Reading Time: An initial **2 minutes** to view **BOTH** sections

MATHEMATICS METHODS : UNITS 3 & 4, 2022

Test 1 – (10%)

3.1.7, 3.1.8, 3.1.10 to 3.1.16, 3.2.1 to 3.2.3, 3.2.6, 3.2.7

PM

Time Allowed	First Name	Surname	Marks
20 minutes	MARKING GUIDE		20 marks

Circle your Teacher's Name: Mrs Alvaro, Mrs Bestall, Mrs Fraser-Jones, Mr Gibbon, Mrs Greenaway, Mr Koulianos, Mr Luzuk, Mrs Murray, Mr Tanday

Assessment Conditions: ***(N.B. Sufficient working out must be shown to gain full marks)***

- Calculators: Not Allowed
- Formula Sheet: Provided
- Notes: Not Allowed

PART A – CALCULATOR FREE

QUESTION 1 **(6 marks)**

a) Differentiate $y=\frac{5x+2}{x\sqrt{x}}$ (do not simplify beyond positive indices). (2 marks)

$$y=\frac{5x+2}{x^{\frac{3}{2}}}$$

$$y'=\frac{5x^{\frac{3}{2}}-\frac{3}{2}x^{\frac{1}{2}}(5x+2)}{x^3}$$

OR

$$y=(5x+2)x^{-\frac{3}{2}}$$

$$y'=5x^{-\frac{3}{2}}+\left(-\frac{3}{2}x^{-\frac{5}{2}}\right)(5x+2)$$

$$=\frac{5}{x^{\frac{3}{2}}}-\frac{3(5x+2)}{2x^{\frac{5}{2}}}$$

✓ Attempts use of product or quotient rule

✓ Correct derivative with positive indices

OR

$$y=5x^{-\frac{1}{2}}+2x^{-\frac{3}{2}}$$

$$y'=-\frac{5}{2x^{\frac{3}{2}}}-\frac{3}{x^{\frac{5}{2}}}$$

✓ Splits numerator

✓ Correct derivative with positive indices

b) Let $g(x)=(2-x^3)^3$.

i) Evaluate $g''(1)$. (3 marks)

$$g'(x)=3(2-x^3)^2(-3x^2)$$
$$=-9x^2(2-x^3)^2$$
$$g''(x)=-18x(2-x^3)^2+2(2-x^3)(-3x^2)(-9x^2)$$
$$=-18x(2-x^3)^2+54x^4(2-x^3)$$
$$g''(1)=36$$

✓ First derivative

✓ Second derivative

✓ Correct value

ii) What does your result in part (i) represent? (1 mark)

The rate of change of the first derivative when $x=1$.

✓ Exact wording

QUESTION 2 **(4 marks)**

The radius of a sphere increases from 10cm to 10.1cm. Find the approximate change in surface area that this causes.

$$S=4\pi r^2$$
$$\frac{dS}{dr}\approx 8\pi r$$
$$r=10$$
$$\delta r=0.1$$
$$\delta S\approx\frac{dS}{dr}\delta r$$
$$\approx 8\pi r\times\frac{1}{10}$$
$$\approx 8\pi$$

There is an increase of approximately 8π cm^2.

✓ Derivative

✓ Values of r and δr

✓ Substitutes into increments formula

✓ States change

QUESTION 3 **(5 marks)**

a) Find $\int\left(5\sqrt{x}+1\right)dx$ (2 marks)

$$\int 5x^{\frac{1}{2}}+1.dx = 5\times\frac{2}{3}+x+c$$

$$=\frac{10x^{\frac{3}{2}}}{3}+x+c$$

✓ Fractional indices

✓ Correct integral

NB: Deduct 1 mark over the question if missing +c

b) Find $\int\left(\frac{t^4-2t^3+1}{2t^2}\right)dt$ (2 marks)

$$\int \frac{t^4}{2t^2}-\frac{2t^3}{2t^2}+\frac{1}{2t^2}.dt = \int\frac{t^2}{2}-t+\frac{t^{-2}}{2}.dt$$

$$=\frac{t^3}{6}-\frac{t^2}{2}-\frac{t^{-1}}{2}+c$$

$$=\frac{t^3}{6}-\frac{t^2}{2}-\frac{1}{2t}+c$$

✓ Splits numerator

✓ Correct integral (positive indices)

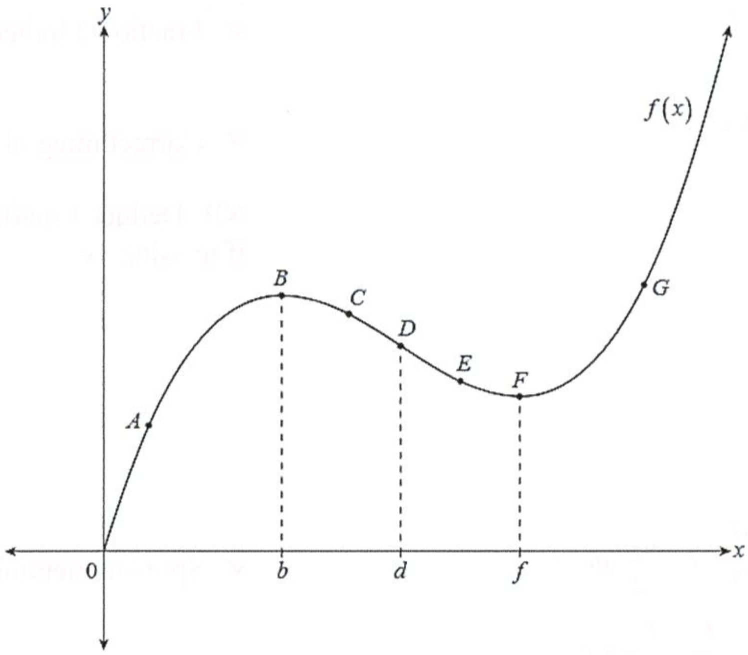
c) Find $\int\left(6(x^4+2x^3)^5(4x^3+6x^2)\right)dx$ (1 mark)

$(x^4+2x^3)^6+c$

✓ Correct integral

QUESTION 4 **(5 marks)**

Consider the function $f(x)$, defined for $x \geq 0$. The graph of $y = f(x)$ is shown below. Point B is a local maximum with x-coordinate b, point D is an inflection point with x-coordinate d, and point F is a local minimum with x-coordinate f.



a) Identify the point(s) (i.e. A, B, C, D, E, F or G) with the following properties:

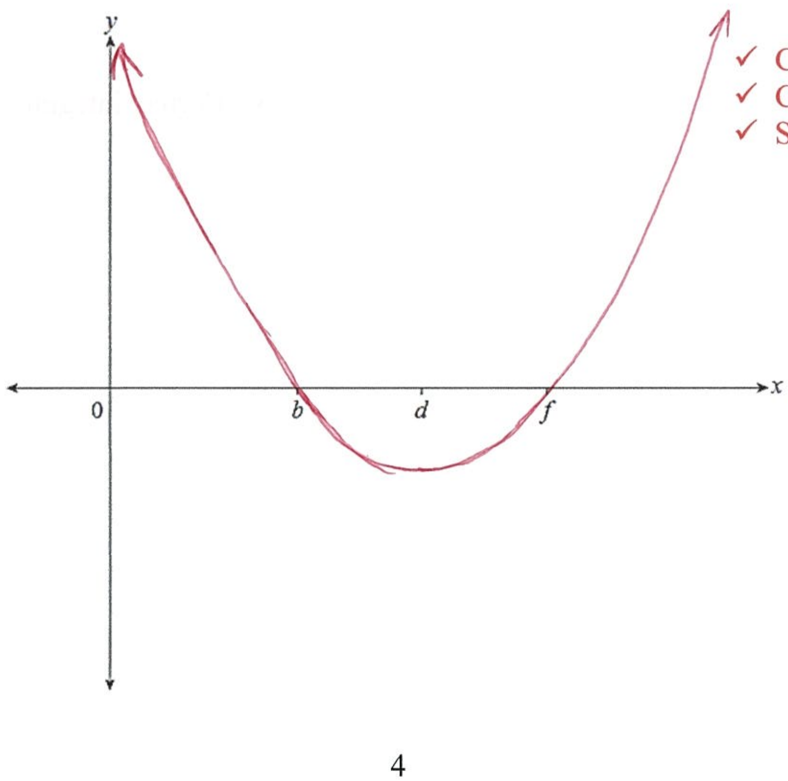
i) $f'(x) < 0$ and $f''(x > 0)$. (1 mark)

E

ii) $f'(x) < 0$ and $f''(x < 0)$. (1 mark)

C

b) On the axes below sketch the graph of $f'(x)$. (3 marks)



✓ Correct x-int
✓ Correct concavity
✓ Smooth curve